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Dependence of the Transverse Thermal Conductivity of Unidirectional Composites on Fiber Shape

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The model for the traverse thermal conductivity of unidirectional composite materials published by Springer and Tsai, which yields good results when compared to experimental results, is generalized to include fibers having elliptical cross sections. It is shown that the thermal conductivity of the unidirectional composite normal to the filaments depends strongly on fiber shape.

KEY WORDS: transverse thermal conductivity; unidirectional composites; fiber shape.

1. INTRODUCTION

To study the transverse thermal conductivity of unidirectional composite materials, in 1967 Springer and Tsai [1] published a short but interesting paper in which they tackled the problem from two different approaches. In the first approach, recognizing the analogy between this thermal conductivity problem and the unidirectional composite response to a shear loading problem, the results by Adams and Doner [2] originally published for shear loading were adopted, which essentially is a finite difference numerical approach. To compare with the shear loading analogy results for thermal conductivity, they developed this so-called thermal model. In Fig. 1, in the region intercepted by the fiber (inner region), the heat flux is treated like a series arrangement taking into consideration the fiber surface curvature. This heat flux result is combined with the heat flux obtained from the regions not affected by the presence of the fibers (outer region). In a simple algebraic exercise, one can establish that in a rectangular fiber

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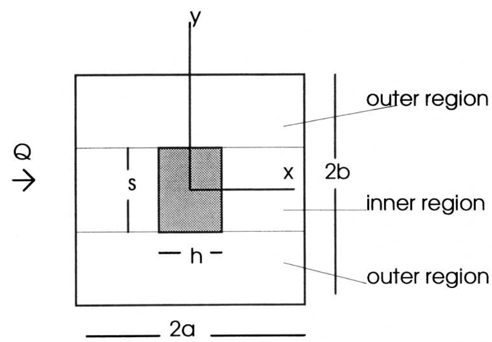

Fig. 1. Rectangular fiber cross section.

case, the thermal model results approach the series results for laminae if the height of the filament approaches the height of the unit cell. Springer and Tsai also examined a circular cross section fiber in their thermal model. When they compared the thermal model with the shear loading analogy model, they claimed that the thermal model predicted a slightly lower value, about 5% lower for a fiber volume fraction less than 75%, and about 10% lower for a fiber volume fraction greater than 75%.

Then, when they compared their theoretical predictions with the experimental results from Ref. 3, they found that for a high thermal conductivity ratio ($k_f/k_m = 666$, where k_f is the fiber transverse thermal conductivity and k_m is the matrix thermal conductivity, assumed to be isotropic), the data agree reasonably well with the results of the shear loading analogy model but are higher than the values predicted by the thermal model. The data also indicate that the assumption of cylindrical fibers is more appropriate than that of square fibers. The differences between the shear analogy model and the thermal model can be understood in that for the shear analogy model, a more accurate two-dimensional temperature distribution is assumed taking into account the heat flux continuity conditions normal to the fiber surface which represent the true physical description; however, the thermal model certainly lacks these conditions. Actually, the temperature is only a function of the distance, in this case x, (Fig. 1) and the normal heat flux continuity condition across the boundary fares well only at the height $y = 0$, but the condition begins to deteriorate as y moves away from the center (Fig. 3 of Ref. 1). Nevertheless, the thermal model still remains a fairly reliable and simple model.

We are also encouraged by the recent experimental work of Rogers et al. [4] who have established the inverse linear relationship of the transverse electrical resistivity to the aspect ratio of ribbon-shaped carbon fibers.

Since the strong inverse correlation of electrical resistivity and thermal conductivity is well established, they concluded that the fiber transverse thermal conductivity should increase further as their aspect ratio increases. Motivated by all the evidence, we would like to extend the thermal model to different fiber cross sections, and hopefully we will be able to predict quantitatively how the transverse composite thermal conductivity depends on the fiber shape. In the following, the derivation is straightforward, and the nomenclature from Ref. 1 is adopted.

2. GENERALIZED THERMAL MODEL

In Ref. 1, Eq. (8), it was shown that the transverse composite thermal conductivity k_{22} can be expressed as

$$\frac{k_{22}}{k_m} \cong \left(1 - \frac{s}{2b}\right) + \frac{a}{b} \int_0^s \frac{dy}{(2a-h) + (kh_m/k_f)} \tag{1}$$

where $2a$ and $2b$ are the dimensions of the unit cell or the filament spacings in x and y directions. k_m and k_f are the thermal conductivities of the matrix and fiber, respectively. The parameter s is the maximum dimension of the filament in the y direction, and h is the width of the filament at a given height y (Figs. 1 and 2).

Rectangular Fiber. For a rectangular fiber cross section, assuming the third dimension is infinitely long, Eq. (1) can be reduced to

$$\frac{k_{22}}{k_m} \cong \left(1 - \frac{s}{2b}\right) + \frac{2a}{2b} \frac{s}{(2a-h) + (hk_m/k_f)} \tag{2}$$

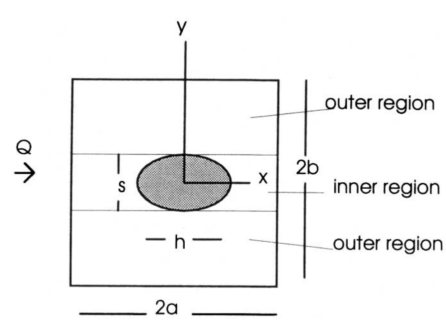

Fig. 2. Elliptical fiber cross section.

Equation (2) can be expressed as the two following equivalent forms:

$$\begin{aligned} 2bk_{22} &= (2b-s)\,k_m + sk_s \\ \frac{2a}{k_s} &= \frac{2a-h}{k_m} + \frac{h}{k_f} \end{aligned} \tag{3}$$

We recognize that Eq. (2) really is a combination of two components, as shown in Eq. (3). First, we combine the thermal conductivity of the fiber and matrix in series fashion to arrive at the value of k_s; then we combine k_s and k_m in parallel fashion to arrive at the final k_{22}. Of course when s approaches $2b$, k_{22} is just k_s, which was referred to as laminae in the series model of Ref. 1.

In Ref. 1, Springer and Tsai further let $a=b$ for a square packing array and $s=h$ for a square filament and expressed Eq. (2) in terms of volume fraction, $v_f = sh/4ab$; thus, a simple expression was obtained. Then, the form of Eq. (2) (i.e., not involving volume fraction) is kept and we point out that the thermal model for a rectangular (including square) fiber case really is a compound model analogous to an electric circuit. Furthermore, we show numerically the dependence of composite thermal conductivity on the shape of fiber cross section by varying the ratio of fiber height to width as a function of volume fraction.

Elliptical Fiber. For a fiber of elliptical cross section (Fig. 2), the shape can be represented by the following equation, where $h=2\alpha$, $s=2\beta$, and α and β are the semiaxes in the x and y directions, respectively. The area of the ellipse is equal to $\pi\alpha\beta$ which is kept constant.

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1 \tag{4}$$

The resulting transverse composite thermal conductivity k_{22}, in a similar fashion as the circular fiber, can be expressed as

$$\frac{k_{22}}{k_m} = \left(1 - \frac{\beta}{b}\right) + \frac{a\beta}{b\alpha B}\left[\pi - \frac{4}{\sqrt{1-\dfrac{B^2\alpha^2}{4a^2}}}\tan^{-1}\frac{\sqrt{1-\dfrac{B^2\alpha^2}{4a^2}}}{1-\dfrac{B\alpha}{2a}}\right] \quad \frac{B^2\alpha^2}{4a^2} \leqslant 1 \tag{5}$$

$$\frac{k_{22}}{k_m} = \left(1 - \frac{\beta}{b}\right) + \frac{a\beta}{b\alpha B}\left[\pi - \frac{2}{\sqrt{\dfrac{B^2\alpha^2}{4a^2}-1}}\log\left|\frac{B\dfrac{\alpha}{2a} - 1 + \sqrt{\dfrac{B^2\alpha^2}{4a^2}-1}}{B\dfrac{\alpha}{2a} - 1 - \sqrt{\dfrac{B^2\alpha^2}{4a^2}-1}}\right|\right] \quad \frac{B^2\alpha^2}{4a^2} \geqslant 1 \tag{6}$$

where

$$B \equiv 2\left(\frac{k_m}{k_f} - 1\right) \tag{7}$$

Of course, if $\alpha = \beta = r$, the radius of a circle, then Eq. (5) becomes Eq. (10) of Ref. 1, provided we express Eq. (5) in terms of volume fraction $v_f = \pi r^2/4a^2$, and $a = b$ for square packing array. In general, Eq. (5) is suitable to use when the fiber thermal conductivity is greater than the matrix thermal conductivity; otherwise Eq. (6) should be used. Based on Eq. (7), it is not hard to see that for the case where k_f is greater than k_m, B^2 is always less than 4, and α is always less than a. The whole combination implies that the argument inside the square root is positive in Eq. (5). On the other hand, if k_m is much greater than k_f, then B is unbounded which implies that the quantity inside the square root is always positive in Eq. (6). Incidentally, we found an error in Ref. 1, namely, the "+" in the denominator of the arc tangent argument in Eq. (10) should be "−."

3. FIBER SHAPE DEPENDENCE

The easiest way to illustrate the composite transverse thermal conductivity dependence on fiber shape, following Ref. 1, is to plot k_{22}/k_m versus different shape factors holding the area constant for different fiber volume fractions, and for different fiber-to-matrix conductivity ratios k_f/k_m for a

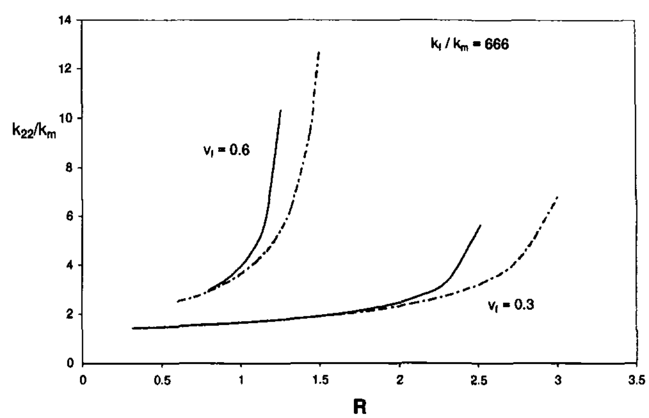

Fig. 3. Composite transverse thermal conductivity for a high ratio of k_f/k_m. The solid curves are for the elliptical fiber cross sections and the dash-dotted curves are for the rectangular fiber cross sections.

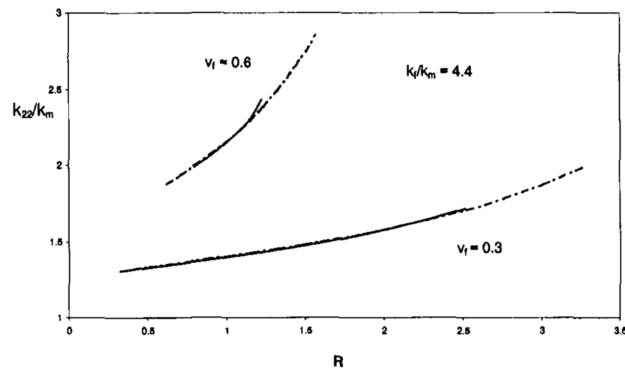

Fig. 4. Composite transverse thermal conductivity for a low ratio of k_f/k_m. The solid curves are for the elliptical fiber cross sections and the dash-dotted curves are for the rectangular fiber cross sections.

few representative cases (Figs. 3–5). The k_f/k_m ratios of 666 and 4.4 used in Ref. 1 were adopted here. Furthermore, the k_f/k_m ratio of 0.2 is arbitrarily chosen, and Eq. (6) is used to obtain Fig. 5. Without losing any generality, the following computations make use of the square packing array arrangement. We further define a variable $R = \alpha/\beta$ for the elliptical fiber case, and $R = h/s$ for the rectangular fiber case. All figures clearly show the strong dependence of k_{22}/k_m on R.

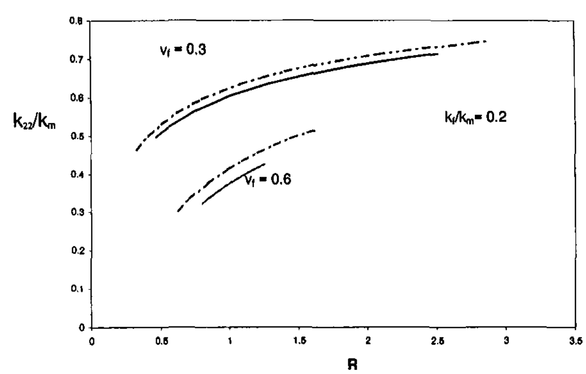

Fig. 5. Composite transverse thermal conductivity for a ratio of $k_f/k_m < 1$. The solid curves are for the elliptical fiber cross sections and the dash-dotted curves are for the rectangular fiber cross sections.

4. CONCLUSION

A general expression has been derived and quantified for composite transverse thermal conductivity for unidirectional elliptical and rectangular cross section fibers. The results are rather interesting in that the composite transverse thermal conductivity can be somewhat manipulated according to the shape of the fiber cross section which may bear some consequence in thermal management. Of course, in reality, there are no truly elliptical fibers in existence, and even if they were to exist, controlling the fiber orientation would be a major processing challenge. The problem could have been formulated in a slightly different fashion, e.g., the transverse thermal conductivity could have been formulated as a function of the fiber orientation angle with respect to the flow direction while maintaining the fiber elliptical shape. However, the two different treatments would give identical results. This simple model presented here is capable of quantifying the relationship between the composite transverse thermal conductivity and fiber shape or its orientation.

NOMENCLATURE

2a, 2b	fiber spacing in x and y directions
B	dimensionless parameter, $B \equiv 2((k_m/k_f) - 1)$
Q	average heat flux per unit cell length
h	width of rectangular filament
s	height of rectangular filament
β	semiaxis of ellipse in y direction
α	semiaxis of ellipse in x direction
v_f	volume fraction $= hs/4ab$ for rectangular fiber $= \pi\alpha\beta/4ab$ for elliptical fiber
k_{22}	composite thermal conductivity in the direction normal to the fibers
k_m	thermal conductivity of matrix
k_f	thermal conductivity of fiber in the direction normal to the fibers
R	h/s ratio of width to height of rectangle
R	ratio of length of semiaxis in x direction to length of semiaxis in y direction of ellipse

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